## Common and Natural Logarithms

## Common Logarithms

- A common logarithm has a _base of 10
- If there is no base given explicitly, it is common.
- You can easily find common logs of powers of ten.
- You can use your calculator to evaluate common logs.


## Natural Logarithms

- A natural logarithm has a $\qquad$ base of $e$ $\qquad$ .
- We write natural logarithms as $\qquad$ ln $\qquad$ .
- In other words, $\log _{e}(1)=\ln (1)$
- If $\ln \mathrm{e}=\mathrm{x}, \quad \log _{e} e=x \quad e^{x}=e \quad e^{x}=e^{1} \quad x=1 \quad \ln \mathrm{e}=1$


## The Number $e$

- The mathematical constant $\boldsymbol{e}$ is the unique real number such that the value of the derivative (the slope of the tangent line) of the function $f(x)=e^{x}$ at the point $x=0$ is exactly 1 .
- The function $\mathrm{e}^{\mathrm{x}}$ so defined is called the exponential function.
- The inverse of the exponential function is the natural logarithm, or logarithm with base e.
- The number $e$ is also commonly defined as the base of the natural logarithm (using an integral to define the latter), as the limit of a certain sequence, or as the sum of a certain series.
- The number $e$ is one of the most important numbers in mathematics, alongside the additive and multiplicative identities 0 and 1 , the constant $\underline{\pi}$, and the imaginary number $i$.
- $\quad e$ is irrational, and as such its value cannot be given exactly as a finite or eventually repeating decimal. The numerical value of $e$ truncated to 20 decimal places is:


## Change of Base Formula

- Allows us to convert to a different base.
- If $a, b$, and $n$ are positive numbers and neither $a$ nor $b$ is 1 , then the following equation is true.

$$
\log _{a} n=\frac{\log _{b} n}{\log _{b} a}
$$

## Examples

