

Houston, We Have a Problem Lesson – Rocketry Handout

Background Information

Hundreds of years ago, the idea of flight did not exist. With the understanding of the basics of flight, inventors determined how to put a man in the air for a few seconds. In the 60s, John F. Kennedy proposed that within a decade, we would put a man on the moon; the United States succeeded with the help of a visionary team and aerospace engineers at NASA.

Today, with the Mars' rover dominating the news from NASA, have you ever wondered what goes into the planning for such an event? While we might not be able to launch our rocket to Mars, we can explore some of the fundamental concepts that govern such a mission. Essentially, there are four forces at play: lift, thrust, gravity and drag. Some forces are more dominant than others, and by understanding what forces are acting on your rocket and the equations governing the forces, you can begin to model the flight of your rocket. Teamwork is essential during this activity, just as it is in professional engineering.

Important Parameters

When beginning to develop your flight plan, the *mass* of your rocket, *cross-sectional area* of the rocket, *thrust* and *impulse* of the engine, and *mass* of the engine all come in to play when calculating the forces acting on the rocket and determining your flight time. Before beginning to calculate equations, review the above parameters. (Note: Remember to keep everything in SI units.)

- **Mass** – The mass of the rocket and engine in grams will need to be converted to kilograms for the calculations. These values are listed in the tables provided.
- **Cross-sectional Area** – This parameter is important when calculating the drag on the object. Large objects have larger surface area and larger drag. The diameter of the rocket is provided and you simply need to determine the area in m^2 .
- **Thrust** – Thrust is the amount of force the engine can put out in Newtons (N).
- **Impulse** – Impulse is the area under the thrust vs. time graph, giving you a value of Newtons*Seconds.

If you know the maximum thrust of the engine and total impulse of the engine (from the tables at end of this handout), how long will the engine burn if it was at maximum thrust for the entire duration of the flight?

Answer: Half as long as shown in the data table: (“Average” total impulse) = (Maximum thrust) * $\frac{1}{2}$ (thrust duration)

Or, if avg thrust = $\frac{1}{2}$ maximum thrust: Impulse = $\frac{1}{2}$ maximum thrust * time or
Impulse = average thrust * time

Note: Rocket goes from zero thrust to a maximum thrust. We assume this happens linearly for sake of calculation.

Stages of Flight

With these parameters defined, break down various sections of your flight.

- **First (boost) Stage** – The rocket is under power from the engine. Drag, thrust, lift, and gravity are acting on the rocket in a delicate balance. The thrust and lift of the rocket are able to overcome the effects of gravity and drag as it flies into the air.
- **Second (coast) Stage** – The engine burns out and now the rocket continues coasting upward. What forces are now acting on the rocket? Gravity and drag are still present but thrust has disappeared, and lift is beginning to diminish as the rocket slows down.
- The rocket reaches the peak of its flight.
- **Third (descent) Stage** – This is the point where your rocket begins its free fall back to Earth. Do you have a parachute? What is the delay after the engine burn out before your parachute deploys? Is the time delay sufficient to reach to apogee (peak) of the flight? How does the parachute affect the path down back to Earth?

Lesson Objectives

- Recognize the various stages of flight for a rocket, along with the forces present in each stage.
- Pick the appropriate rocket and engine size to get to a height.
- Utilize the equations governing the flight of a rocket as it goes through various stages of flight. Have a basic understanding of the important parameters that govern the flight of the rocket (see bolded variables).
- Independently research a topic and effectively work in groups to construct a rocket.

Equations

You have an idea of what parameters affect flight and an idea of the forces present at different stages of the flight, so how do you determine how high your rocket will go? The general idea is balancing of forces, for which everything should relate back to $F = m \cdot a$. The equations necessary for this lesson are summarized below.

(Note: Visit <http://my.execpc.com/~culp/rockets/RocketEquations.pdf> to understand the derivations of the equations.)

Variables

a = acceleration (m/s^2)

A = rocket cross-sectional area in (m^2)

c_d = drag coefficient = 0.75 for average rocket (unitless)

F = force in ($kg \cdot m/s^2$)

g = acceleration of gravity = $9.81 m/s^2$

h_B = altitude at burnout in (m)

h_C = coasting distance in (m)

I = motor impulse in (N*s)

m_E = engine mass (including propellant) in (kg)

m_p = propellant mass in (kg)

m_R = rocket mass in (kg)

q and p are intermediate variables needed to solve later equations in m/s and s-1 respectively

T = "AVERAGE" motor thrust in Newton (kg*m/s²)

v_T = burnout velocity in (m/s)

ρ = air density = 1.223 kg/m³

τ = thrust duration or motor burn time in seconds

$m_b = m_R + m_E - m_p/2$ Mass present in first (boost) stage (kg)

$m_c = m_R + m_E - m_p$ Mass present in second (coast) stage (kg)

$k = \frac{1}{2} * \rho * C_d * A$ Air Drag coefficient (kg/m)

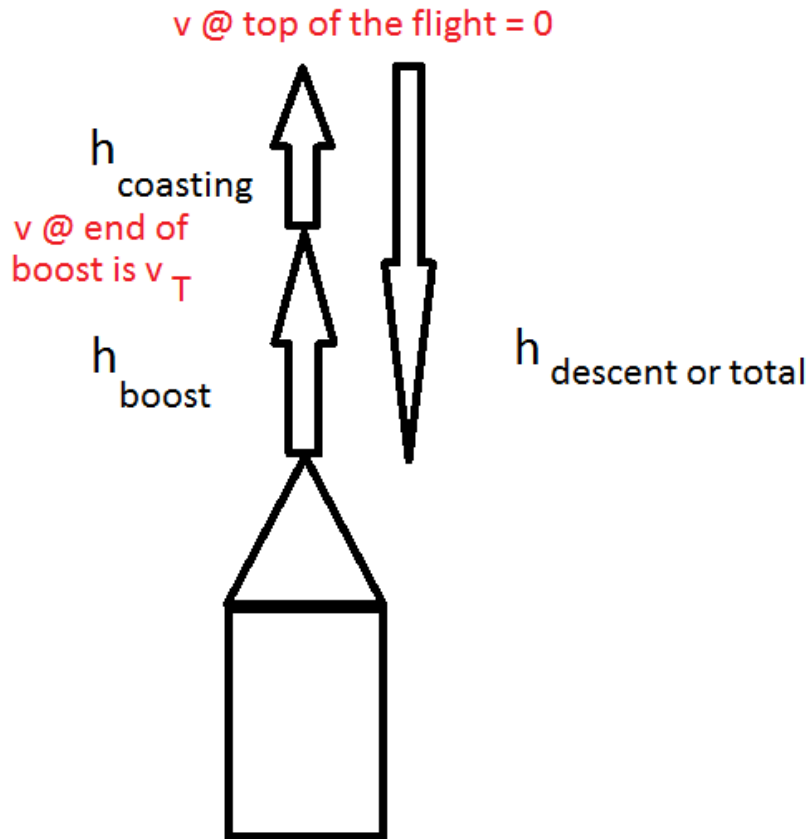
Forces

$F_{g,b} = m_b * g$ Force acting on the rocket due to gravity during First (boost) Stage

$F_{g,c} = m_c * g$ Force acting on the rocket due to gravity during Second (coast) Stage

$F_{Drag} = k * v_T^2$ Force acting on the Rocket due to drag

$F_{Thrust} = T$ Thrust from the rocket



Equations: Stage 1

(Note: see figure above for a better understanding of the variables)

Looking at the tables provided below, can you determine what values will need to be calculated?

The velocity at the time of burnout will certainly be something that you will need to calculate. It can be done by equating the forces acting on the rocket to $m \cdot a$, or the total force acting on the rocket. If we consider that $a = \frac{\Delta v}{\Delta t}$, we can calculate the following for the velocity at end of the burnout (pay close attention to the bolded variables as those directly apply to your current knowledge of physics; others derived by the calculus will only be solved algebraically):

$$m \cdot \frac{\Delta v}{\Delta t} = T - m_b \cdot g - k \cdot v_\tau^2 \quad \text{Net force} = ma = \text{Thrust} - \text{Force due to gravity} - \text{Drag}$$

$$v_\tau = q \cdot \frac{1 - e^{-p \cdot \tau}}{1 + e^{-p \cdot \tau}} \quad (1) \quad v_\tau = \text{velocity at the end of burnout, where:}$$

$$q = \sqrt{\frac{T - m_b \cdot g}{k}} \quad (2) \quad q = \text{intermediate value}$$

$$p = \frac{2 \cdot k \cdot q}{m_b} \quad (3) \quad p = \text{intermediate value}$$

But how does this relate back to height reached in the first stage? If additional calculus is applied, h_b can be determined as follows:

(Note: If the derivations are beyond your knowledge, algebra can be used to determine the boost height with the following equation.)

$$h_b = \frac{m_b}{2k} * \ln \left(\frac{q^2}{q^2 - v_t^2} \right) \quad (4) \quad h_b = \text{boost height}$$

You know all of the variables in this equation and can find the height of your flight under the power of the engine.

Is this the point where it reaches the apogee? Answer: *No, we still have to take into account how high it will coast after the rocket burns out.*

Equations: Stage 2

The rocket's engine burns out, but has enough momentum to continue its upward trajectory. For this stage, you still have drag and gravity acting on the rocket but the mass has changed due to burning of all the fuel, and there is no more thrust from the engine. Equating the forces acting on the rocket to $m \cdot a$, you can get the following equation for the height achieved under no thrust (focus only on the resulting algebraic equation to determine the coast height):

$$h_c = \frac{m_c}{2k} * \ln \left(\frac{q_c^2 - v_t^2}{q_c^2} \right) \quad (5) \quad h_c = \text{coast height}$$

where:

$$q_c^2 = \frac{-m_c \cdot g}{k} \quad (6) \quad q_c^2 \text{ is an intermediate calculus value}$$

The total height reached is simply:

$$h_T = h_b + h_c \quad (7) \quad h_T = \text{total height}$$

Team Proposal

Submit a team proposal for launching a rocket. This is to include your mission objective, rocket and engine selection, and all calculations performed. Should your proposal be approved, you will build and launch your rocket.

Team Members (roles for the duration of the project):

- **Flight Director** coordinates the proposal, and then ensures that everything is going smoothly and other members are performing their tasks.
- **Parts Marshal** ensures the correct parts are ordered and assembled
- **Mission Commander** launches the rocket

Rockets Selection

Note: All Rockets, with the exception of Hi-Flier, are mini rockets and require mini engines (Standard rockets require standard engines). The mini engines listed are 1/4A3-3T, 1/2A3-2T, A3-4T, A10-3T. The rest are standard engines.

Available Rockets	Weight (g)	Diameter (mm)
Bandito	17	19
Firestreak	31.2	22
Quark	3	14
Swift	2.5	14
Hi-Flier	23.2	19
Star-Trooper	8.5	14
Mosquito	3.1	14

Engine Selection

Engine Type	Total Impulse	Time Delay	Max Lift Wt. (g)	Max Thrust (N)	Thrust Duration (s)	Initial Weight (g)	Propellant Weight (g)
1/4A3-3T	0.625	3	28	4.9	0.25	5.6	0.85
1/2A3-2T	1.25	2	57	8.3	0.3	5.6	1.75
A3-4T	2.5	4	57	6.8	0.6	7.6	3.5
A10-3T	2.5	3	85	13	0.8	7.9	3.78
1/2A6-2	1.25	2	57	8.9	0.3	15	1.56
A8-3	2.5	3	85	10.7	0.5	16.2	3.12
B4-2	5	2	113	13.2	1.1	19.8	8.33
B4-4	5	4	99	13.2	1.1	21	8.33
B6-2	5	2	127	12.1	0.8	19.3	6.24
B6-4	5	4	113	12.1	0.8	20.1	6.24

Note: Numbers labeled as “weights” in these tables are actually masses.