Name: ____ Date: _________________ Class: ______________

Mouse Trap Racer Analysis Worksheet ANSWER SHEET

Accelerometer

The acceleration sensor we are using outputs dimensionless values in the range -400 to 400. To convert these values to readings in acceleration units m/sec^2, use the following formula

$$a = \frac{sensor\ output}{200} \times 9.80\ m/sec^2$$

For example, suppose the accelerometer is reading a value of 145 units. The corresponding acceleration to three significant figures

$$a = \frac{145}{200} \times 9.80\ m/sec^2 = 7.11\ m/sec^2$$

Exercise 1

In the NXT Mindstorms software, open and read the help menu for the **Acceleration Sensor Block**.

- How many axes are available for applications? __________ Answer: Three
- Which axis are you using for the mouse trap racer? __________ Answer: The x axis
- What wire color is used to carry number data? __________ Answer: Yellow (look at Figure 4 or the program in file mousetrap.rbt)
- Draw a diagram of the way you oriented your accelerometer. → Answer: The correct way is shown in Image 3

Acceleration, velocity, and displacement

The mouse trap racer starts at rest. Therefore, our initial velocity is 0 m/sec. On launch, a torque is applied to the rear axle by the wound string attached to the mouse trap bracket. Assuming sufficient traction between the floor and wheel tires, a nonzero force is applied to the racer. According to Newton's second law, this causes the racer to accelerate in the direction of the force. Assuming equal force, a racer of the smallest mass will have the largest acceleration, as expressed by the formula

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

We see that the mass and acceleration ratios are inversely related.

Suppose that between the given time instants t_1 and t_2, the racer is accelerating with constant acceleration a_1. Acceleration can be defined as a change in velocity divided by the interval between these two time instants t_1 and t_2

$$a_1 = \frac{v_2 - v_1}{t_2 - t_1}$$

Setting $t_1 = 0$ and $v_1 = 0$, and solving for v_2 we obtain

$$v_2 = a_1 t_2$$

as the velocity at time t_2. Next, suppose that between time instants t_2 and t_3, the racer is accelerating with constant acceleration a_2. Accelerations a_1 and a_2 are **not** necessarily equal. Therefore we have

$$a_2 = \frac{v_3 - v_2}{t_3 - t_2}$$

Solving for v_3, we obtain

$$v_3 = a_2(t_3 - t_2) + v_2$$

and substituting for v_2, we get

$$v_3 = a_2(t_3 - t_2) + a_1 t_2$$

Continuing in this way, we can obtain a formula for velocity v_4 at time t_4

$$v_4 = a_3(t_4 - t_3) + a_2(t_3 - t_2) + a_1 t_2$$

and so on for v_5, v_6, v_7.... As the string runs out, the racer starts to decelerate due to friction forces between the axles and chassis, and acceleration becomes negative, and we should remember to figure in acceleration terms with a negative sign.

Having computed the velocity versus time from the acceleration versus time data, let us now compute displacement versus time. Velocity can be defined as the change in position divided by the time interval. For the time interval $t_2 - t_1$ and constant velocity v_1, we have

$$v_1 = \frac{x_2 - x_1}{t_2 - t_1}$$

Setting $t_1 = 0$ and $x_1 = 0$, and solving for x_2 we obtain

$$x_2 = v_1 t_2$$

as the distance the racer has traveled from time zero to time t_2. Next, suppose that between time instants t_2 and t_3, the racer is again traveling with constant velocity v_2. Velocities v_1 and v_2 are **not** necessarily equal. Therefore we have

$$v_2 = \frac{x_3 - x_2}{t_3 - t_2}$$

Solving for x_3, we get

$$x_3 = v_2(t_3 - t_2) + x_2$$

and substituting for x_2, we get the total distance traveled at time t_3

$$x_3 = v_2(t_3 - t_2) + v_1 t_2$$

The distance traveled from $t_1 = 0$ to t_4 is similarly

$$x_4 = v_3(t_4 - t_3) + v_2(t_3 - t_2) + v_1 t_2$$

and so on for x_5, x_6, x_7.... Finally, since the mouse trap racer will always move forward, all velocities will be positive.

In the above, we assumed that acceleration and velocity values did not change between the given times. However, we can guess that these values do change, and in fact, change **continuously** between the times that accelerometer readings are taken. We therefore ask

- **How can we better model the fact that acceleration and velocities change continuously and still be able to use our formulas from above?**

One simple way to answer this question is to take the **average** of acceleration and velocity values between the reading times. For example, instead of assuming that the constant acceleration over the interval t_1 to t_2 is a_1, we assume that this acceleration is $\frac{1}{2}(a_1 + a_2)$, and similarly for the velocity values. In this way, we are able to form a better guess (or estimate) of these values. For other reading times, we form similar averages. For example, over the interval t_3 to t_4, we will use the acceleration estimate $\frac{1}{2}(a_3 + a_4)$ and the velocity estimate $\frac{1}{2}(v_3 + v_4)$. So given the accelerometer readings a_1, a_2, a_3, … at the times t_1, t_2, t_3, …, we can compute the velocity at any time by substituting acceleration average $\frac{1}{2}(a_i + a_{i+1})$ for a_i and velocity average $\frac{1}{2}(v_i + v_{i+1})$ for v_i in the formulas we derived above.

Exercise 2

- Derive an expression for v_8, the velocity at time t_8 using average accelerations between each reading time (**Hint**: first work out the formula for v_8 up to time t_8, and then substitute $\frac{1}{2}(a_i + a_{i+1})$ for a_i, for $i = 1$ to $i = 7$).

  Answer: First we compute

  $v_8 = a_7(t_8-t_7) + a_6(t_7-t_6) + a_5(t_6-t_5) + a_4(t_5-t_4) + a_3(t_4-t_3) + a_2(t_3-t_2) + a_1t_2$

  Then we substitute average accelerations for the given ones

  $v_8 = (a_7+a_8)(t_8-t_7)/2 + (a_6+a_7)(t_7-t_6)/2 + (a_5+a_6)(t_6-t_5)/2 + (a_4+a_5)(t_5-t_4)/2 + (a_3+a_4)(t_4-t_3)/2 + (a_2+a_3)(t_3-t_2)/2 + (a_1+a_2)t_2/2$

- Derive an expression for x_8, the total displacement at time t_8 using average velocities between each reading time (**Hint**: in your answer to first question, substitute velocity for acceleration on the right side, and displacement for velocity on the left side).

  Answer: Substituting we get

  $x_8 = (v_7+v_8)(t_8-t_7)/2 + (v_6+v_7)(t_7-t_6)/2 + (v_5+v_6)(t_6-t_5)/2 + (v_4+v_5)(t_5-t_4)/2 + (v_3+v_4)(t_4-t_3)/2 + (v_2+v_3)(t_3-t_2)/2 + (v_1+v_2)t_2/2$

- Suppose the accelerometer readings are given in units of m/sec^2 as 4, 3, 2, 1, 0, -2, -1, 0. The corresponding times are given in seconds as 0, 1, 2, 3, 4, 5, 6, 7. Compute the velocity versus time and distance traveled versus time (**Hint**: substitute these numbers into the equations you found above).

  Answer: The times are $t_1 = 0$, $t_2 = 1$ and so on. Velocities are $v_1 = 4$, $v_2 = 3$ and so on. Substituting and computing we get

  $v_2 = a_1t_2$
  $v_3 = v_2 + a_2(t_3-t_2)$
  $v_4 = v_3 + a_3(t_4-t_3)$
  $v_5 = v_4 + a_4(t_5-t_4)$
  $v_6 = v_5 + a_5(t_6-t_5)$
  $v_7 = v_6 + a_6(t_7-t_6)$
  $v_8 = v_7 + a_7(t_8-t_7)$

  So

  $v_1 = 0$
  $v_2 = 4$
  $v_3 = 4 + 3 = 7$
  $v_4 = 4 + 3 + 2 = 9$
  $v_5 = 4 + 3 + 2 + 1 = 10$
  $v_6 = 4 + 3 + 2 + 1 + 0 = 10$
  $v_7 = 4 + 3 + 2 + 1 + 0 – 2 = 8$

$v_8 = 4 + 3 + 2 + 1 + 0 - 2 - 1 = 7$

For distance we get

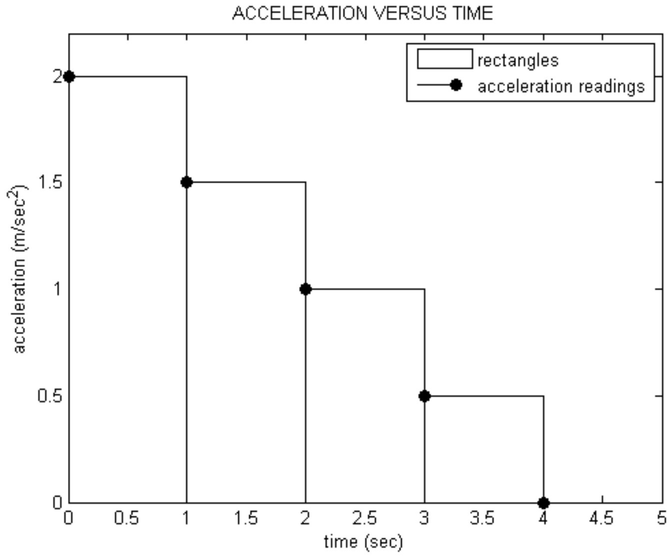
$x_2 = v_1 t_2$
$x_3 = x_2 + v_2(t_3 - t_2)$
$x_4 = x_3 + v_3(t_4 - t_3)$
$x_5 = x_4 + v_4(t_5 - t_4)$
$x_6 = x_5 + v_5(t_6 - t_5)$
$x_7 = x_6 + v_6(t_7 - t_6)$
$x_8 = x_7 + v_7(t_8 - t_7)$

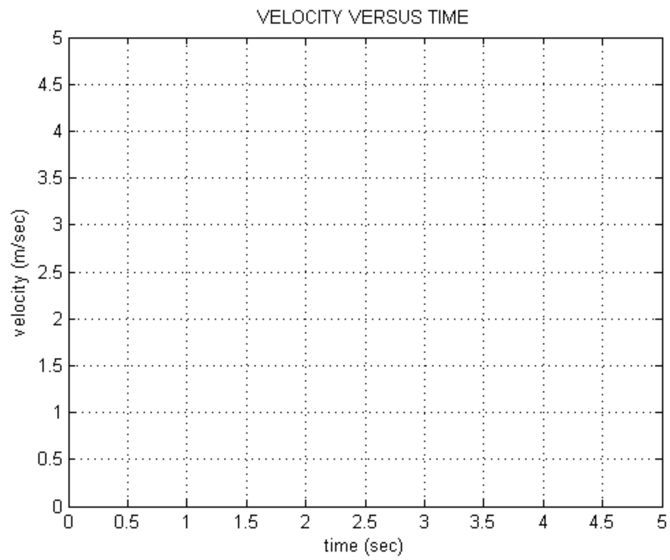
So

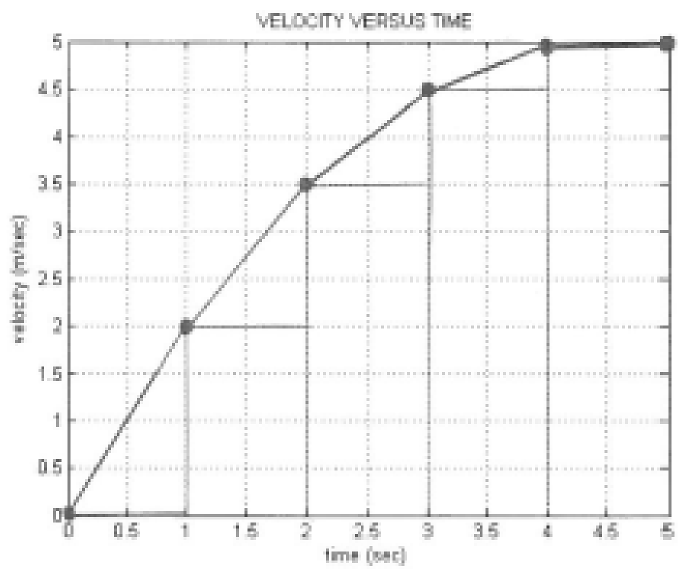
$x_1 = 0$
$x_2 = 4$
$x_3 = 4 + 7 = 11$
$x_4 = 4 + 7 + 9 = 20$
$x_5 = 4 + 7 + 9 + 10 = 30$
$x_6 = 4 + 7 + 9 + 10 + 10 = 40$
$x_7 = 4 + 7 + 9 + 10 + 10 + 8 = 48$

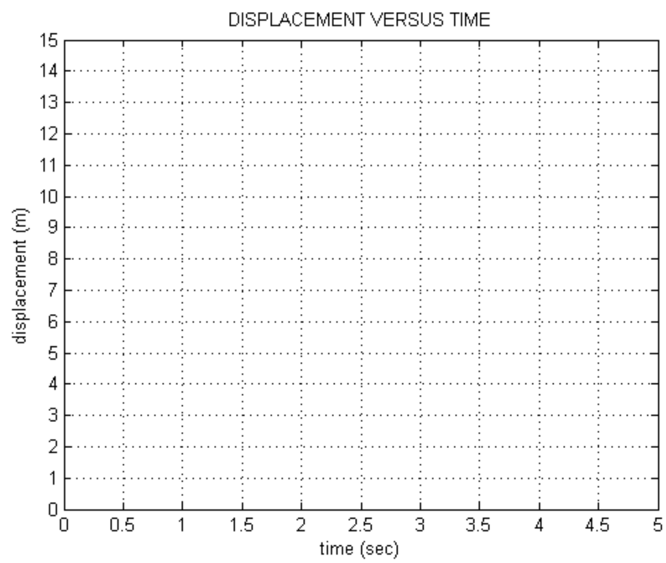
Using average accelerations/velocities between times is similar

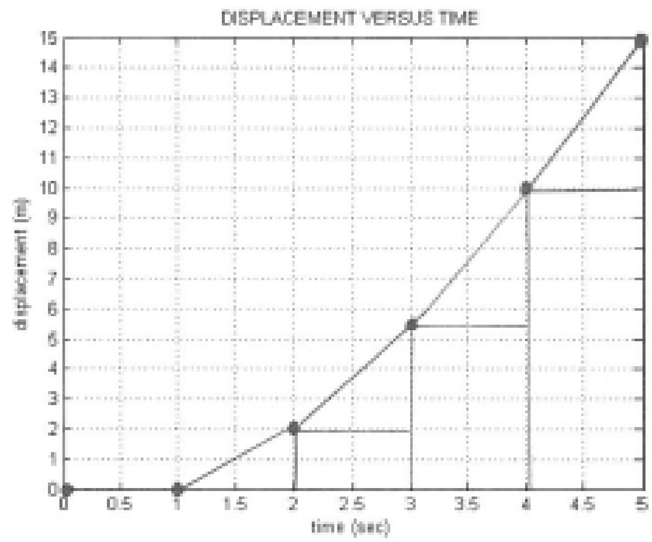
- Now the accelerometer readings are plotted on a graph using black circles below. Notice that the area of the first rectangle with height 2 is equal to 2, and so on for the other rectangles. Sketch graphs for the velocity versus time, and displacement versus time, assuming the racer is initially stationary. Use the provided empty axes.




VELOCITY VERSUS TIME
velocity (m/sec)
time (sec)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


VELOCITY VERSUS TIME
velocity (m/sec)
time (sec)





- What happens as the difference between the time instants is decreased – in other words, as we start taking readings at a larger and larger rate?

  Answer: The acceleration, velocity, and displacement values will change more frequently, and eventually it will become impossible to tell whether they change in a piecewise constant fashion or continuously.

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