

A Frictional Roller Coaster Project Rubric

The purpose of this **engineering design challenge project** is to apply differential calculus, physics, and numerical calculations to design a simple two-dimensional roller coaster for which the friction force is considered, and build a model using



basic materials like foam pipe wrap insulation and marbles. The roller coaster path must be made from differentiable functions, such that the piecewise curve created is differentiable at all points.

Using the work-energy theorem and considering the work done by friction along the roller coaster path, student teams mathematically verify that their designs are functional—that is, a marble must be able to go through the entire path of the roller coaster, beginning and ending with velocity equal to zero. Then students build physical models to test that their designs really work.

Deliverables: 1) calculations and graphs in Excel, 2) a presentation-report including the design process and calculations that verify design functionality, and 3) working physical roller coaster model.

Engineering Challenge Project Guidelines

1. Work in groups of three or four.
2. Design a roller coaster path using at least five differentiable functions. Parabolas are suggested because the standard form equation allows you to place the vertex at the desired height and translate them easily. The challenge is to join the parabolas, one upward-opening and one downward-opening, such that the new curve is differentiable at the joint point.
3. Your design dimensions must be appropriate to the flexibility of the material you use to build the model; foam pipe insulation with 1.5-inch external and 0.5-inch internal diameters. That means, no very sharp curves or loops.
4. Use the velocity formula obtained from the work-energy theorem applied to a body rolling on an incline with friction to determine the maximum height a marble will reach in the upward-opening parabolas. Then the vertices of the downward-opening parabolas may be defined based on this maximum height. Use this formula to mathematically support the efficiency of your design: velocity along the path must be more than zero. Use an Excel spreadsheet to compute the necessary calculations.
5. The conditions for this project are: initial marble's velocity at the beginning of the path equals zero, final velocity at the end of the path also must be zero (or almost). Your design must be efficient enough that the initial potential energy of the body (marble) is enough to make it complete through the entire path.
6. Graph the piecewise function defining your designed path, and the calculated velocities along the path.
7. Build a physical model using the foam pipe insulation. Your model must match your design dimensions.
8. To hold your physical model, use a cardboard backing, masking tape and vinyl supports (see the Appendix). Make sure the marble you use has a diameter that is less than the internal diameter of the pipe insulation. You may use different materials, but inform the teacher before making any changes/additions.
9. Test your model at least 10 times and record the results. Make conclusions about your design and model. *Is the model behaving as expected? If not, why? What were the failures? How did you solve them?*
10. Create a slideshow (or video) to present the different steps of your work: design process, calculations done, construction and model built. This report must include overall analysis of the work done, problems found, and conclusions. See the checklist and rubrics below.
11. Make a class presentation of your results and constructed/tested model.

Image source: (Takabisha roller coaster in Japan) 2012 Alex Brogan, Wikimedia Commons
CC BY-SA 3.0 https://commons.wikimedia.org/wiki/File:Takabisha_roller_coaster.jpg

Checklist

Final Project Results Presentation-Report	Max Points	Evaluation
1. Slide with project title and student names	5	
2. Slide with project description and purpose, and a summary of the results obtained	5	
3. Slide(s) that describe the curves used to create piecewise roller coaster path, and method used to obtain a continuous and differentiable path.	20	
4. Slide(s) with the graph and final explanation of the piecewise roller coaster path.	5	
5. Slide(s) that describe how the work-energy theorem was used to verify design functionality and to find the velocity of the body along the roller coaster path.	20	
6. Slide(s) with the graphs of the velocity of the body, coefficient of friction, and friction force versus horizontal displacement; graphs must be labeled, and x-y-axes units specified.	5	
7. Slide(s) that describe the construction process, materials used, final dimensions, accuracy with respect to mathematical design, and photos of the constructed roller coaster model.	15	
8. Slide(s) that explain the prototype tests performed and results obtained.	10	
9. Slide(s) with final conclusions about how similar the expected results were from the mathematical design, and the final prototype results.	10	
10. Slide(s) with professionally presented bibliography-references used.	5	
Total Points	100	

In-Class Results Presentation Checklist	Max Points	Evaluation
11. Slideshow or video (mp3, wma, mpeg) containing information and results listed in items 1-10 (above) with recorded explanations. A full report delivered as a standalone slideshow or video with recorded explanations earns 40 extra points.	20 or 60	
12. In-class results presentation with proficient presentation and answers.	30	
13. Students look professional (as if for a professional job interview).	10	
Total Points	100	

Notes:

- No projects accepted after the due date:
- In-class project-presentation is mandatory.
- During tutorial time, help will be available for the physics concepts, Excel calculations and graphs, physical model construction, and presentation.
- **Due date: [____ date here ____]**



Image source: (Anaconda roller coaster in VA) 2007 Checho16, Wikimedia Commons
 CC BY-SA 3.0 https://commons.wikimedia.org/wiki/File:Anaconda_roller_Coaster.jpg

Additional Resources

Calculus

Larson, Ron, Edwards, Bruce, and Hostetler, Robert P. *Calculus of a Single Variable*. Eighth Edition. Boston, MA: Houghton-Mifflin, 2006.

Briggs, William L., Cochran, Lyle, and Gillett, Bernard. *Calculus AP Edition*. Upper Saddle River, NJ: Pearson Education, 2014.

Physics

Young, Hugh D., & Freedman, Roger. *University Physics with Modern Physics*. 14th Edition, Pearson, 2016.

Harris, Benson. *University Physics*. Second Edition, John Wiley & Sons, Inc., 1995.

Alonso, Marcelo, & Edward, Finn. *Fundamental University Physics, Volume 1 Mechanics*. Addison-Wesley, 1966 (Spanish version: *Fisica Volumen 1, Mecanica*. Fondo Educativo Interamericano, 1976).

Halliday, David, & Resnick, Robert. *Physics, Parts 1 and 2 Combined*. Second Edition. John Wiley & Sons, Inc., 1966 (Spanish version: *Fisica, Edicion Combinada Partes I y II*, CECSA, Tercera Edicion, 1976).

General Mathematics

Demana, Franklin, et al. *Precalculus, Graphical, Numerical, Algebraic*. Second Edition. Pearson, 2016.

Pownal, Malcom. *Functions and Graphs, Calculus Preparatory Mathematics*, Prentice-Hall, 1983.

Excel and PowerPoint

Excel: Get Started with Formulas and Functions. Microsoft Office Support.

<https://support.office.com/en-us/article/Get-Started-with-Formulas-and-Functions-e0b10c56-700c-4961-a7b2-a0fc5866735e>

Tips for Making Effective PowerPoint Presentations. May 28, 2009. National Conference of State Legislatures. <http://www.ncsl.org/legislators-staff/legislative-staff/legislative-staff-coordinating-committee/tips-for-making-effective-powerpoint-presentations.aspx>

Microsoft PowerPoint Tutorials. Electric Teacher. <http://www.electricteacher.com/tutorial3.htm>

Project Support

AP Calculus First Semester Project online tutorial for students at Sophia:

<https://www.sophia.org/playlists/ap-calculus-first-semester-project>, on the following topics:

- Friction concepts and solutions for the problem of a body rolling on a surface with friction
- Rotational kinematics and dynamics
- Piecewise differentiable functions

Roller Coasters

Brooks, Meade. *Physics Concepts in Action / Physics Roller Coaster*. 2014. Collin College, Frisco, TX. <http://iws.collin.edu/mbrooks/student%20research/projects/Rollercoaster%20Simulation/index.html>

Roller Coaster Physics. Real-World Physics Problems.com. <http://www.real-world-physics-problems.com/roller-coaster-physics.html>

How Do Roller Coasters Work? Wonder of the Day #1239. Physical Science, Science, Wonderopolis.org. National Center for Families Learning. <http://wonderopolis.org/wonder/how-do-roller-coasters-work>

Sastamoinen, Shawna. *Roller Coaster Physics: The Science Behind the Thrills*. 2002. Physics 211X, University of Alaska Fairbanks. http://ffden-2.phys.uaf.edu/211_fall2002.web.dir/shawna_sastamoinen/Roller_Coasters.htm

Final Project Results Presentation-Report Grading Rubric

	Below Standard	Meets Standard	Above Standard
Project Title, Student Names	<ul style="list-style-type: none"> - Incomplete or missing project name, student names, and/or date 	<ul style="list-style-type: none"> - Project name, student names, and date displayed - Professional text/graphics on title slide 	<ul style="list-style-type: none"> - Project name, student names, date displayed - Pleasing graphic and or animation - Background music
Project Description	<ul style="list-style-type: none"> - Missing or incomplete outline of the project objective - Missing or incomplete outline of the procedures and resources used - Missing or incomplete outline of obtained results 	<ul style="list-style-type: none"> - Correct description of project objective - Correct outline of the procedures and resources used - Correct summary of obtained results 	<ul style="list-style-type: none"> - Correct description of project objective - Correct outline of the procedures and resources used - Correct summary of obtained results - Figures/images that help understanding - Eye-catching slide format and/or animations
Roller Coaster Path Design	<ul style="list-style-type: none"> - Incomplete or missing description of the differentiable functions used to create the piecewise path - Fewer than 5 differentiable functions used to create the piecewise path - Incomplete or missing process used to create the piecewise path from the differential functions - Incomplete or missing final functional expression for the piecewise path - Incomplete or missing roller coaster path graph - Text incorrectly formatted, labeled or separated - Text difficult to read (font size < 24, color not enough contrast with slide background) 	<ul style="list-style-type: none"> - Complete description of the differentiable functions used to create the piecewise path - At least five differentiable functions used to create the piecewise path - Complete process used to create the piecewise path from the differential functions - Complete final functional expression for the piecewise path - Complete and well-formatted roller coaster path graph - Text formatted, clearly labeled and separated - Text easy to read (font size > 24, color good contrast with background color) 	<ul style="list-style-type: none"> - Complete description of the differentiable functions used to create the piecewise path - At least five differentiable functions used to create the piecewise path - Complete process used to create the piecewise path from the differential functions - Complete final functional expression for the piecewise path - Complete and well-formatted roller coaster path graph - Text formatted, clearly labeled and separated - Text easy to read (font size > 24, font color good contrast with background color) - Eye-catching slide format and animations
Mathematical Design Tested Using the Work-Energy Theorem	<ul style="list-style-type: none"> - Incomplete or missing description of the analysis of the roller coaster path functionality using the work-energy theorem - Incorrect or missing mathematical expression used in the project for the work-energy theorem - Incomplete or missing graphs of the velocity of the body, friction coefficient, and friction force along the piecewise path 	<ul style="list-style-type: none"> - Complete description of the analysis of the roller coaster path functionality using the work-energy theorem - Correct mathematical expression used in the project for the work-energy theorem - Axes correctly formatted and labeled - Complete graph set of the velocity of the body, friction coefficient, and friction force along the piecewise path - Graph titles included and easy to read - Graph colors with good contrast 	<ul style="list-style-type: none"> - Complete description of the analysis of the roller coaster path functionality using the work-energy theorem - Correct mathematical expression used in the project for the work-energy theorem - Axes correctly formatted and labeled - Complete graph set of the velocity of the body, friction coefficient, and friction force along the piecewise path - Graph titles included and easy to read - Graph colors with good contrast - Eye-catching slide format and animations.

Name: _____

Date: _____

Class: _____

	Below Standard	Meets Standard	Above Standard
Prototype's Construction Process and Final Prototype	<ul style="list-style-type: none"> - Incomplete or missing construction process description - Incomplete or missing materials list for building the physical prototype - No photo of the final prototype - Missing/incorrect prototype scale factor - Missing/incomplete analysis of the accuracy of the mathematical model and prototype - Text difficult to read (font size <24, font color not enough contrast with slide background) 	<ul style="list-style-type: none"> - Complete construction process description - Complete materials list for building the physical prototype - Photo of the final prototype - Correct prototype scale factor provided - Analysis of the accuracy of the mathematical model and prototype - Text easy to read (font size >24, font color contrasts with background color) 	<ul style="list-style-type: none"> - Complete construction process description - Complete materials list for building the physical prototype - Several photos show the final prototype and the construction process - Correct prototype scale factor provided - Analysis of the accuracy of the mathematical model and prototype - Text formatted, clearly labeled and separated - Text displayed easy to read (font size >24, font color contrasts with background color) - Eye-catching slide format and animations
Prototype Tests	<ul style="list-style-type: none"> - Missing or incomplete prototype tests - Fewer than 10 tests run - Incomplete/missing testing results obtained - Incomplete/missing test conclusions - Incomplete/missing analysis of observed deviations in the prototype behavior and the expected behavior - Text difficult to read - Text colors difficult to see 	<ul style="list-style-type: none"> - Complete prototype tests - At least 10 tests run - Complete testing results obtained - Tests conclusions provided - Analysis of observed deviations in the prototype behavior and expected behavior - Text easy to read (font size >24, font color contrasts with background color) 	<ul style="list-style-type: none"> - Complete prototype tests - 10 or more tests run - Complete testing results obtained - Tests conclusions provided - Analysis of observed deviations in the prototype behavior and expected behavior - Text formatted, clearly labeled and separated - Text displayed easy to read (font size >24, font color contrasts with background color) - Eye-catching slide format and animations.
Roller Coaster Prototype Construction	<ul style="list-style-type: none"> - Unfinished roller coaster prototype, not fully functional and/or missing - Prototype shape does not match original design - Prototype built at incorrect scale 	<ul style="list-style-type: none"> - Finished and fully functional prototype - Prototype shape matches original design - Prototype built to correct scale - Neat and tidy prototype finishing 	<ul style="list-style-type: none"> - Finished and fully functional prototype - Prototype shape matches original design - Prototype built to correct scale - Neat and tidy prototype finishing - Artistic touches in prototype finishing
Project Conclusions, and Bibliography-References	<ul style="list-style-type: none"> - Missing, incomplete or incorrect conclusions - Conclusions missing or poorly done in problem context - Missing/incomplete references and source materials list; poorly presented 	<ul style="list-style-type: none"> - Correct conclusions, procedures and results - Conclusions completed in problem context - Complete references/source materials that are professionally presented 	<ul style="list-style-type: none"> - Correct conclusions, procedures and results - Conclusions completed in problem context - Complete references/source materials that are professionally presented - Eye-catching slide formats, animations and figures that aid in audience understanding

In-Class Results Presentation Grading Rubric

	Unprofessional	Nearly Professional	Professional
Body Language	<ul style="list-style-type: none"> - Sight not on the audience - Reads from notes or slides - Lacks confidence during the entire presentation - Distracting, unnatural movements or gestures (fidgeting or nervous) 	<ul style="list-style-type: none"> - Some audience eye contact - Sometimes reads from notes or slides - Some confidence and poise (but still appears nervous) - Some movement and gestures 	<ul style="list-style-type: none"> - Eye contact with audience - Little or no reading from slides or notes - Looks confident during the entire presentation - Natural movements, gestures
Voice	<ul style="list-style-type: none"> - Speaks too softly to be understood and/or heard - Speaks too quickly or slowly - Frequently uses words or sounds like: Okay, so..., you know..., uh, umm, I mean... - Does not use correct technical language or formal English 	<ul style="list-style-type: none"> - Speaks clearly most of the time - Sometimes speaks too quickly or slowly - Speaks loudly enough for most of the audience - Occasionally uses words or sounds like: Okay, so..., you know..., uh, umm, I mean... - Uses correct technical language or formal English most of the presentation 	<ul style="list-style-type: none"> - Speaks clearly during the entire presentation - Speaks at uniform volume, and at a normal pace, not too quickly or slowly - Speaks loudly enough for everyone to hear - Rarely or never uses words or sounds like: Okay, so..., you know..., uh, umm, I mean... - Uses correct technical language or formal English during the entire presentation
Overall Presentation	<ul style="list-style-type: none"> - No main idea present, wrong, or incomplete - Ideas presented in a wrong and/or illogical sequence - Missing important steps in the development - Missing, incorrect or incomplete introduction and/or conclusion, - Poor presentation time management - Incorrect answers to questions 	<ul style="list-style-type: none"> - Main idea present, but not proficiently explained - Ideas presented in a logical order but not proficiently connected or missing important points - Introduction and conclusion, present but are not effective - Presentation completed in the allotted time, but time not proficiently distributed over topics or ideas - Correctly answered most questions, and in context 	<ul style="list-style-type: none"> - Main idea presented in a clear and effective way - Ideas presented in a logical order, emphasizing main points, and in context - Effective introduction and conclusion - Presentation completed in the allotted time, and time proficiently distributed over topics or ideas - Correctly answered questions, and in context; also enriched answers with relevant info and/or pertinent examples
Students Look	<ul style="list-style-type: none"> - Wearing inappropriate clothes for the occasion 	<ul style="list-style-type: none"> - Wearing semi-formal clothes 	<ul style="list-style-type: none"> - Wearing appropriate clothes for the occasion, as if for a professional job interview

Appendix: Suggested Materials for Physical Roller Coaster Model

For this project, it is recommended that you build the rollercoaster model using inexpensive materials that are easy to work with. Other materials than those suggested below may be used, but notify the teacher in advance. The following materials can be found in hardware and craft supply stores such as Home Depot, Lowes, Office Depot, Hobby Lobby or Michaels.

Suggested fabrication materials:

- foam pipe wrap insulation, Tubolit ½-in x 6-ft
- vinyl bullnose archway corner bead, ¾-in x 10-ft
- corrugated cardboard or plastic sheets, 3 x 4-ft size
- 1-inch masking tape
- glass marbles

Additional tools and resources:

- yardstick or 1-m ruler with inch-scale markings
- scissors, utility knife, super glue
- (optional) pegboard, 2 x 4-ft, to use as a *mat* to easily draw points on cardboard sheet

